Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

September 25, 2023

Entrance Exam (TSE)
Physics

Duration: 1H30

Exercise 1	**(12 points)**

A mechanical oscillator consists of a block (S) of mass m and a spring of negligible mass and force constant k = 20 N/m. The spring is connected from one of its ends to a fixed support A. (S) is attached to the other end of the spring and it may slide without friction on a horizontal support (figure 1).

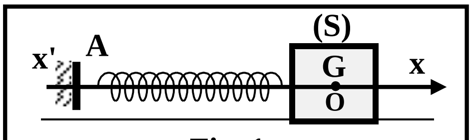

Fig. 1

At equilibrium, G, the center of mass of (S), coincides with the origin O of the x-axis.

At the instant $t_0 = 0$, G is at O and we launch (S) with a velocity $\vec{V}_0 = V_o\,\vec{i}$; thus, (S) undergoes mechanical oscillations with an amplitude X_m.

At an instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic value of its velocity is $v = x' = \frac{dx}{dt}$.

The aim of this exercise is to study for this oscillator the effect of v_0 on the oscillation amplitude X_m.

Take:

- the horizontal plane passing through G as a reference level for gravitational potential energy;
- $g = 10\ m/s^2$ and $\pi^2 = 10$.

1) Theoretical study

1.1) Write the expression of the mechanical energy ME of the system (Oscillator, Earth) in terms of x, m, k and v.

1.2) Determine the second order differential equation that governs the variation of x.

1.3) Deduce the expression of the proper (natural) period T_0 of the oscillations in terms of m and k.

2) Experimental study

An appropriate device gives the elastic potential energy Epe of the oscillator as a function of time for two different experiments, experiment 1 and experiment 2 (figure 2).

2.1) Use the graphs of figure 2 in order to:

2.1.1) Justify that the oscillations of (S) are undamped.

2.1.2) Copy and then complete the following table:

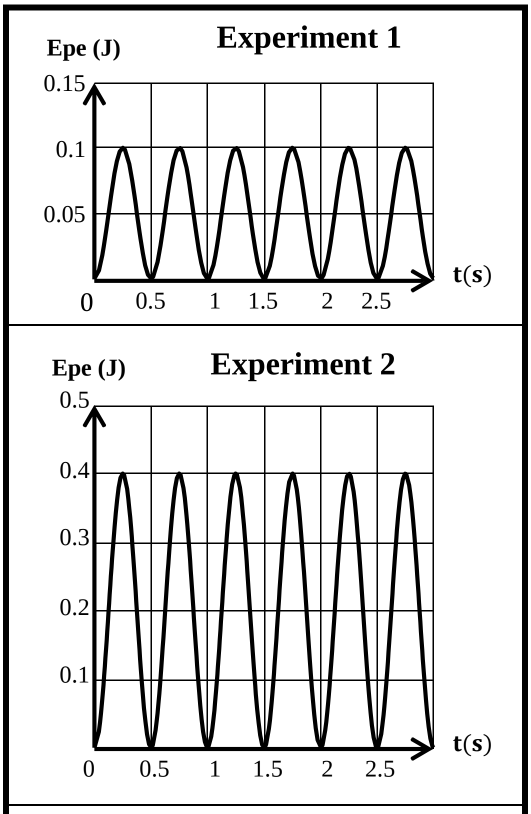

Fig. 2

	Experiment 1	Experiment 2
The maximum value of Epe		
The value of the period T_E of Epe		

2.2) Show that m = 0.5 kg knowing that $T_0 = 2T_E$.

2.3) Show that $X_{m(2)} = 2\ X_{m(1)}$, where $X_{m(1)}$ and $X_{m(2)}$ are the amplitudes of the oscillations in experiments 1 and 2 respectively.

2.4) Determine the values of v_0 for the two experiments.

Exercise 2	**(12 points)**

Charging and discharging of a capacitor

The aim of this exercise is to determine, by two different methods, the value of the capacitance C of a capacitor. For this aim, we set-up the circuit of figure 3. This circuit is formed of an ideal generator delivering a constant voltage of value E = 10 V, a capacitor of capacitance C, two identical resistors of resistances R1= R2 = 10 kΩ and a double switch K.

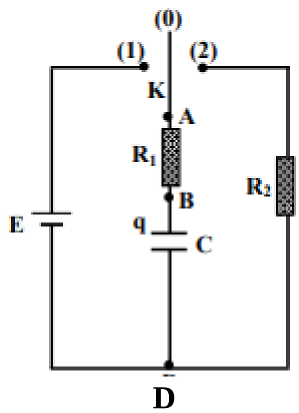

Fig.3

A – Charging the capacitor

The switch K is in the position (0) and the capacitor is neutral.
At the instant $t_0 = 0$, we turn K to position (1) and the charging of the capacitor starts.

<u>Theoretical study</u>

Applying the law of addition of voltages and taking the positive direction along the circuit as that of the current, show that the differential equation that describes the variation of the voltage $u_c = u_{BD}$ across the capacitor has the form:

$$E = R_1 C \frac{du_c}{dt} + u_c$$

1) The solution of this differential equation has the form: $u_c = A(1 - e^{-\frac{t}{\tau_1}})$, where A and τ_1 are constants. Show that $A = E$ and $\tau_1 = R_1C$.

2) Show that at the end of charging $u_c = E$.

3) Show that the expression $u_{AB} = u_{R1} = E\,e^{-\frac{t}{R_1 C}}$.

4) Establish the expression of the natural logarithm of u_{R1} [$\ell n(u_{R1})$] as a function of time.

<u>Graphical study</u>

The variation of $\ell n(u_{R1})$ as a function of time is represented by figure 4.

5) Justify that the shape of the obtained graph agrees with the expression of $\ell n(u_{R1})$ as a function of time.

6) Deduce, using the graph, the value of the capacitance C.

B – Discharging the capacitor

The capacitor being fully charged, we turn the switch K to position (2). At an instant $t_0 = 0$, taken as a new origin of time, the discharging of the capacitor starts.

1) During discharging, the current circulates from B to A in the resistor of resistance R_1. Justify.

2) Taking the positive direction along the circuit as that of the current, show that the differential equation in the voltage u_c across the capacitor has the form:

$$u_c + (R_1 + R_2)C\frac{du_c}{dt} = 0.$$

3) The solution of the above differential equation has the form $u_c = E\,e^{-\frac{t}{\tau_2}}$ where τ_2 is the time constant of the circuit during discharging. Show that $\tau_2 = (R_1 + R_2)C$.

4) The variation of the voltage u_c across the capacitor and the tangent to the curve $u_c = f(t)$ at the instant $t_0 = 0$, are represented in figure 5. Deduce, from this figure, the value of the capacitance C.

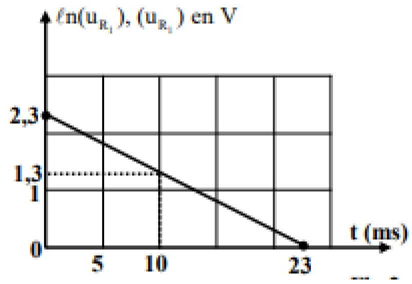

Fig.4

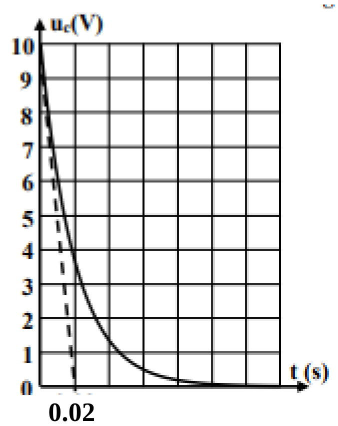

Fig.5

Exercise 3	**(6 points)**

Consider a circular conducting loop of radius r = 10 cm and resistance R = 2 Ω (figure 6). The loop is placed in a uniform magnetic field $\vec{B}$.

During the time interval [0 , 2s], the value B of the magnetic field $\vec{B}$ decreases with time according to the relation: B = - 0.04t + 0.8 (SI).

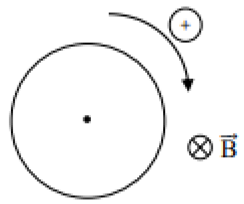

Fig.6

1) Justify that a current is induced in the loop during the time interval [0 , 2s].
2) Apply Lenz's law in order to specify the direction of the induced current.
3) Determine the expression of the magnetic flux crossing the loop as a function of time.
4) Deduce the value of the induced electromotive force e.
5) The current carried by the loop is given by the relation i = e/R. Deduce the value of the current i.