Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

September 15, 2022

Entrance Exam
(Engineering)
Physics

Duration: 1H30

Exercise 1	**(13 points)**

Consider a mechanical oscillator constituted of a spring, of negligible mass, and of un-joined loops of stiffness k and a solid (S) of mass m = 0.1 kg.

The spring, placed horizontally, is fixed from one of its extremities to a fixed support and (S) is attached to the other extremity. (S) may slide without friction on a horizontal rail AB and its center of inertia G can move along a horizontal axis x'Ox. At equilibrium, G coincides with the origin O of the axis x'x (figure 1).

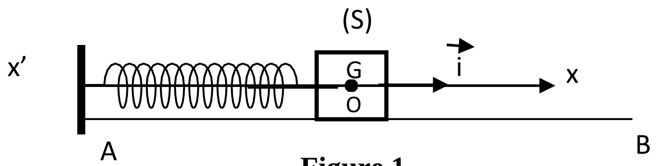


Figure 1

The solid (S) is displaced from its equilibrium position by a distance $x_0 = \overline{OG}_0$ and we give it, at the instant $t_0 = 0$, in the positive direction an initial velocity $\vec{V}_0 = V_0\ \vec{i}$. Thus, (S) performs mechanical oscillations along x'Ox.

Part I - Theoretical study

At the instant t, the abscissa of G is $x = \overline{OG}$ and the algebraic measure of its velocity is $v = \frac{dx}{dt}$.

Take the horizontal plane passing through G as a reference level of gravitational potential energy.

1) Write, at an instant t, the expression of the mechanical energy ME of the system (oscillator, Earth) interms of m, x, k and v.

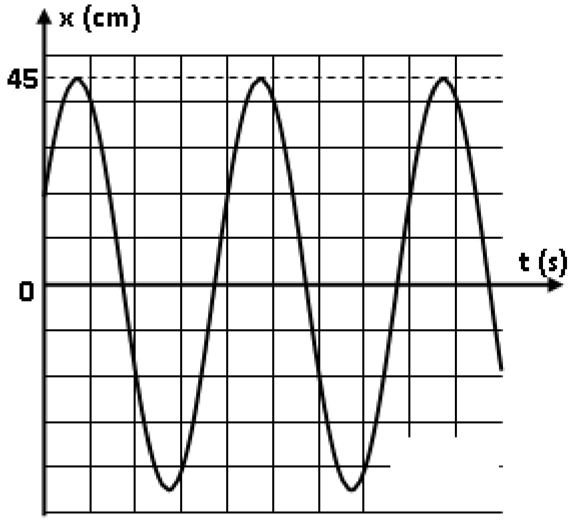
2) Establish the second order differential equation in x that describes the motion of G.

3) The solution of this differential equation has the form: $x = X_m\ sin(\frac{2\pi}{T_0}t + \varphi)$, where X_m, T_0 and φ are constants. Determine the expression of the proper period T_0 in terms of m and k.

Part II - Graphical study of the motion

An appropriate device allows to obtain the variations with respect to time:

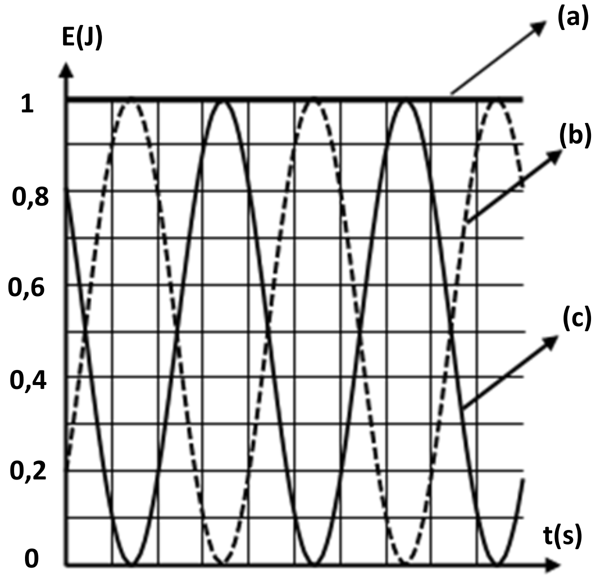
- of the abscissa x of G (figure 2);
- of the kinetic energy KE, of the elastic potential energy PE_e and of the mechanical energy ME of the system (oscillator, Earth) (figure 3).



1 horizontal division → 0.157 s

1 vertical division → 10 cm

Figure 2



2 horizontal divisions → 0.157 s

Figure 3

4) Referring to figure 2, indicate the value of:
a) the initial abscissa x_0;
b) the amplitude X_m;
c) the period T_0.

5) Determine the values of k and φ.

6) The curves (a), (b), and (c) of figure 3 represent the variations of the energies of the system (oscillator, Earth) as a function of time. Using this figure:
a) identify, with justification, the energy represented by each curve;
b) Determine the value of the initial velocity v_0.

Exercise 2 (11 points)

The aim of this exercise is to determine the capacitance C of a capacitor. We set-up the series circuit of the figure 4.

This circuit includes:

- an ideal battery of electromotive force E = 10 V;
- a rheostat of resistance R;
- a capacitor of capacitance C;
- an ammeter (A) of negligible resistance;
- a switch K.

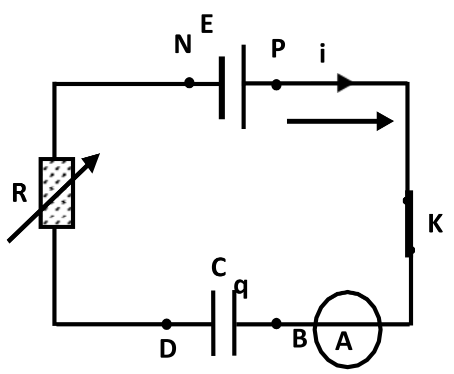


Figure 4

Initially the capacitor is uncharged. We close the switch K at the instant $t_0 = 0$. At an instant t, plate B of the capacitor carries a charge q and the circuit carries a current i as shown in the figure 4.

1) Write the expression of i in terms of C and u_C , where $u_C = u_{BD}$ is the voltage across the capacitor.
2) Establish the differential equation that governs the variation of u_C.
3) The solution of this differential equation is of the form: $u_C = a + b\, e^{\frac{-t}{\tau}}$.
Determine the expressions of the constants a, b and τ in terms of E, R and C.
4) Deduce that the expression of the current is: $i = \frac{E}{R}\, e^{\frac{-t}{RC}}$.
5) The ammeter (A) indicates a value $I_0 = 5$ mA at $t_0 = 0$. Deduce the value of R.
6) Write the expression of $u_R = u_{DN}$ in terms of E, R, C and t.
7) At an instant $t = t_1$, the voltage across the capacitor is $u_C = u_R$.
   a) Show that $t_1 = RC\, \ell n2$.
   b) Calculate the value of C knowing that $t_1 = 1.4$ ms.

Exercise 3	**(6 points)**

A homogeneous metallic rod MN of length ℓ, slides on two horizontal and parallel metallic rails AA' and EE' at a constant velocity $\vec{V}$ (figure 5). During its sliding, the rod remains perpendicular to the rails and its center of mass G moves along the axis Ox.

At the instant $t_0 = 0$, G is at O, the origin of abscissa. At an instant t, the abscissa of G is $x = \overline{OG}$ and $v = \frac{dx}{dt}$ is the algebraic value of its velocity. The whole set-up formed of the rod and the rails is put within a uniform magnetic field $\vec{B}$ perpendicular to the plane of the horizontal rails (figure 5).

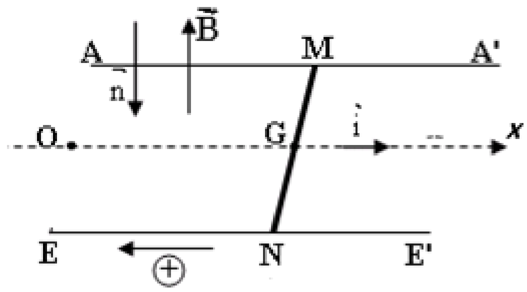


Figure 5

1) Determine, at the instant t, the expression of the magnetic flux crossing the surface AMNE in terms of B, ℓ and x, taking into consideration the chosen arbitrary positive direction on figure 5.
2) Explain the existence of an induced e.m.f e across the ends M and N of the rod.
3) Determine the expression of the induced e.m.f e in terms of B, ℓ and v.
4) No current would pass in the rod. Why?
5) Deduce the polarity of the points M and N of the rod and give the expression of the voltage u_{NM} in terms of e.