Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

Entrance Exam (BC)

Math Exam

September 08, 2025	**(Grade/80)**	**Time: 2 hours**

N.B: All questions are obligatory.

Question 1. (16 points)

The following table represents the demanded quantity y_i of an article, in thousands of objects, as function of the unit price x_i expressed in hundreds of dollars.

Unit price x_i (in hundreds of dollars):	3.5	4	4.5	5	5.5	6
Demanded quantity y_i (in thousands of objects):	140	120	100	95	85	70

1) Represent, in an orthonormal system, the scatter plot associated to the distribution (x_i, y_i).

2) Calculate the coordinates of the center of gravity $G(\overline{X}, \overline{Y})$ and plot it in the same system of reference.

3) Calculate the coefficient of correlation r. Interpret the result obtained.

4) Write the equation of the regression line $(D_{y/x})$ of y in terms of x, then draw this line in the same system.

5) Suppose that this model remains valid till the price of 1500 dollars.

a- Estimate the demanded quantity of this article for a unit price of 700 dollars.

b- Calculate the percentage of decrease in the demanded quantity when the unit price varies from 500 dollars to 700 dollars.

c- Starting from what unit price, the demanded quantity becomes less than or equal to 40000 objects?

6) a- Verify that the elasticity of the demand with respect to the unit price is $E(x) = \frac{26.28\,x}{-26.28\,x+226.52}$

b- Calculate E(5). Is the demand elastic for a unit price of 500 dollars? Interpret the obtained result.

Question 2. (16 points)

In the table below, only one answer among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

N°	**Questions**	**Answers**		
		A	**B**	**C**
1	$\ln(7^3) + \ln(7e^9) + 4\ln\frac{1}{7} - e^{2\ln 3} =$	0	7	e
2	$\lim\limits_{x\to\sqrt{e}} \frac{\ln(x^2) - 1}{x^2 - e} =$	$\frac{1}{e^2}$	$\frac{1}{e}$	$\frac{1}{\sqrt{e}}$
3	If $f(x) = \ln(1 + e^{-x})$, then $f'(x) =$	$\frac{1}{e^x + 1}$	$\frac{-1}{e^{-x} + 1}$	$\frac{-1}{e^x + 1}$
4	The domain of definition of the function h defined by: $h(x) = \ln(e^x - e) + \frac{1}{x-2}$ is:	$]e\,;+\infty[$	$]1;2[\,\cup\,]2;+\infty[$	$]1;+\infty[$

Question 3. (16 points)

In a factory, two machines, A and B, are employed in the production process. The output items are classified into three categories: normal, defective and reworked.

- 80% of the production are defective of which 70% are from machine A.
- 10% of the production are defective of which 80% are from machine A.
- 65% of the production are from machine A.

One item is randomly selected from the production line. Consider the following events:

N: « The selected item is normal »

D: « The selected item is defective »

R: « The selected item is reworked »

A: « The selected item is produced by machine A ».

1) a- Calculate $P(R)$, $P(N \cap A)$ and $P(D \cap A)$.

b- Deduce $P(R \cap A)$ and verify that $P(A/R) = 0.1$.

2) The selected item is produced by machine B. What is the probability that it is defective?

3) Calculate the probability that the selected item is not normal or is from machine B.

4) In this part, suppose that the number of produced items is 200.

Three items are randomly and simultaneously selected from the 200 items.

Calculate the probability of the event E: « None of the three items is normal, at least one item is defective, and at least one item is reworked ».

Question 4. (32 points)

Part A:

Consider the function f defined over $[0; +\infty[$ by: $f(x) = 2x + e^{-2x+1}$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $f(0)$, $f(2)$ and $f(4)$.

2) a- Calculate $\lim\limits_{x \to +\infty} f(x)$, then show that the line (D) of equation $y = 2x$ is an asymptote to (C).

b- Show that (D) is below (C).

3) Calculate $f'(x)$, then set up the table of variation of f.

4) Draw (D) and (C).

5) The equation $f(x) = 4$ admit a unique solution α. Verify that $1.97 < \alpha < 1.99$.

Part B: In the following suppose that $\alpha = 1.98$.

A factory manufactures x hundreds of objects such that $0 < x \leq 10$. The total cost of production is given by: $C_T(x) = 2x^2 + xe^{-2x+1}$ expressed in millions LL.

1) a- Determine the marginal cost $C_m(x)$ as function of x.

b- Calculate $C_m(0.5)$ and then give an economical interpretation of the result.

2) a- Show that the average cost is given by: $C_M(x) = f(x)$.

b- For what produced quantity is the average cost minimal? What is this minimal average cost?

3) The profit function, as function of x, is given by: $P(x) = 4x - 2x^2 - xe^{-2x+1}$.

a- Determine the revenue function R(x) as function of x and deduce the unit selling price if 80% of the manufactured objects are sold.

b- Show that $P(x) = x[4 - C_M(x)]$.

c- Deduce the number of objects to be sold in order that the factory achieves zero gain.