Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

Entrance Exam (BC)

Math Exam

(Grade/80)

September 10, 2024 **Time : 2 hours**

N.B: Choose three questions from the questions 1, 2, 3 and 4.
The question 5 is obligatory.

Question 1. (16 points)

The following table shows the number of views y_i (in thousands), of the first five videos posted by Samir on TikTok.

Rank of the video: x_i	1	2	3	4	5
Number of views (in thousands): y_i	0.9	1.6	4.3	12.1	13.1

1) Represent, in an orthonormal system, the scatter plot associated to the distribution (x_i, y_i).
2) Calculate the coordinates of the mean point $G(\overline{X}, \overline{Y})$ and plot this point in the preceding system.
3) Calculate the coefficient of correlation r. Interpret the result obtained.
4) The equation of the regression line $(D_{y/x})$ of y in terms of x is: $y = mx - 4.07$ where m is a real number. Show, using the results of part 2), that $m = 3.49$, then draw this line in the same system.
5) Suppose that the above pattern remains valid till the 10th video.
   a- Estimate the number of views of the 6th video posted.
   b- In fact, 17870 people watched the 6th video. Calculate the percentage error of the previous estimation.
6) According to reports from top influencers, TikTok pays 0.04$ for every 1000 views. Samir says that, after posting the 7th video, his account balance exceeds 2.8$. Is he right? Justify.

Question 2. (16 points)

In the table below, only one answer among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Answers		
		A	B	C
1	The value of $A = \dfrac{\ln\left(\frac{1}{e}\right)+2\ln\left(e\sqrt{e}\right)}{\ln(2e)-\ln\left(\frac{2}{e^3}\right)}$ is:	2	-1	0.5
2	$\lim\limits_{x\to+\infty} x\ln\left(1+\dfrac{1}{x}\right) =$	1	e^{-1}	$+\infty$
3	The following system of equations: $\begin{cases} e^{x+1} \times e^{y-1} = 2 \\ \ln x + \ln y = \ln(x-1) + \ln(y-1) \end{cases}$ admits:	One solution	No solution	Infinity of solutions
4	The domain of definition of the function h defined by $h(x) = \ln\left(2-\sqrt{x-3}\right)$ is:	$]-\infty\,;7[$	$[3;7[$	$]3;7[$
5	If $C(x) = x^2(\ln(x)-1)$ represents the total cost and x is the number of objects produced $(x > 0)$, then the marginal cost $C_m(x) =$	$\ln(x) - x$	$2x\ln(x)$	$x(2\ln(x)-1)$

Question 3. (16 points)

Consider two urns U and V.

U contains **one** ball carrying number **7** and **three** balls each carrying number **2.**
V contains **three** balls each carrying number **4** and **two** balls each carrying number **2**.

1) One ball is drawn at random from each urn.

a- Calculate the probability of obtaining an odd sum.

b- Show that the probability of obtaining a sum which is a multiple of 3 is $\frac{11}{20}$.

2) In this part, we select one of the two urns:

- If the selected urn is U, then two balls are drawn simultaneously and randomly.
- Otherwise, two balls are drawn successively and without replacement.

Consider the following events:

E: «Urn U is chosen»;
F: «Urn V is chosen»;
G: «The sum obtained is less than 7»;
H: «The sum obtained is less than 6».

a- Calculate $p(G / E)$. Deduce $p(G \cap E)$.

b- Calculate $p(G \cap F)$ and deduce $p(G)$.

c- The sum of the numbers is at least 7. What is the probability that we have chosen urn U?

d- Calculate $p(H)$.

Question 4. (16 points)

Consider the function g defined over $]1; +\infty[$ by: $g(x) = 2x - x^2 - \ln(x-1)$ and let (C_g) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

Part A:

1) Determine $\lim_{x \to 1^+} g(x)$ and $\lim_{x \to +\infty} g(x)$.

2) Show that g is strictly decreasing and set up the table of variations of g.

3) Calculate $g(2)$. Deduce, according to the values of x, the sign of $g(x)$.

Part B:

Consider the function h defined over $]1; +\infty[$ by: $h(x) = \frac{\ln(x-1)}{(x-1)} - x$ and let (C_h) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculer $\lim_{x \to 1^+} h(x)$. Deduce an asymptote to (C_h) .

2) a- Calculate $\lim_{x \to +\infty} h(x)$ and show that the line of equation (d): $y = -x$ is an asymptote to (C_h).

b- Discuss, according to the values of x, the relative position between (C_h) and (d).

3) Show that $h'(x) = \frac{g(x)}{(x-1)^2}$, then set up the table of variations of $h(x)$.

4) Write an equation of the tangent (T) to (C_h) at a point of abscissa $e + 1$.

5) Draw (d), (T) and (C_h).

N.B: the question 5 is obligatory.

Question 5. (32 points)

Part A:

Consider the function f defined over $[0; +\infty[$ by: $f(x) = (x^2 + x)e^{-x+2}$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $f(0)$, $f(1)$ and $f(5)$.

2) Show that (C) is above the axis of abscissas for all values of x in $[0; +\infty[$.

3) Determine $\lim\limits_{x \to +\infty} f(x)$. Deduce an asymptote to (C).

4) Show that $f'(x) = (-x^2 + x + 1)e^{-x+2}$, then set up the table of variations of f.

5) Write an equation of the tangent (T) to (C) at O.

6) Draw (T) and (C).

Part B:

A factory produces certain objects of unit price x expressed in hundreds of millions of LL such that $0.1 \leq x \leq 8$.

The demand and supply functions expressed in thousands of units, are given by: $d(x) = (x + 1)e^{-x+2}$ and $s(x) = \frac{10}{e^{x-1}}$ respectively.

(The unit price is the price of 1000 objects).

1) Calculate the number of demanded objects for a unit price of 400 000 000 LL.

2) Calculate the unit price for a supply of 3260 objects.

3) a- Solve the equation $d(x) = s(x)$.

b- Deduce the equilibrium price in LL, then find the corresponding number of objects.

4) a- Verify that the function $f(x)$ represents the revenue, in hundreds of millions of LL, obtained from selling the demanded quantity with $0.1 \leq x \leq 8$.

b- For what unit price is the revenue maximum? Calculate in this case the price of one object in LL.

5) a- Show that the elasticity of the demand is given by: $E(x) = \frac{x^2}{x+1}$.

b- Calculate $E(1.5)$. Interpret economically the result.

c- The adjacent table represent the table of variation of elasticity $E(x)$.
For what values of x, is the demand elastic? Justifiy.

x	0.1	1.62	8
E(x)	0.01	1	7.11