Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

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Entrance Exam (BC)
Mathematics Exam

September 22, 2023 **Time : 2 hours**

N.B: Choose three questions from the questions 1, 2, 3 and 4 (the question 5 is obligatory).

Question 1. (16 points)

The table below shows an estimation of the number, in thousands, of immigrants from Lebanon (y_i) from July 2022 till December 2022, and the rank of the corresponding month (x_i).

Month	July	August	September	October	November	December
Rank of the month : x_i	1	2	3	4	5	6
Number of immigrants (in thousand): y_i	3	3.5	4.2	5.3	6.5	7.9

1) Determine the percentage increase of the number of immigrants from July till December.
2) Draw, in a rectangular system, the scatter plot of the points associated to the distribution (x_i, y_i).
3) Calculate the coordinates of the mean point $G(\overline{X}, \overline{Y})$ and plot this point in the preceding system.
4) Find the covariance between x and y. Is the relationship between x and y positive or negative?
5) Calculate the linear correlation coefficient r and give an interpretation of its value.
6) Determine an equation of the regression line $(D_{y/x})$ of y in terms of x; draw this line in the preceding system.
7) Suppose that the preceding model remains valid till the end of the year 2023.
   a- Estimate the number of immigrants in February 2023.
   b- Determine the month in which the number of immigrants from Lebanon exceed 11000 for the first time.

Question 2. (16 points)

A factory produces a certain type of objects. The demand function and the supply function are respectively modeled as $D(p)=\dfrac{2}{2+e^{p-1}}$ and $S(p)=\dfrac{2e^{p-1}}{2+e^{p-1}}$; where p is the unit price expressed in millions LL. $D(p)$ and $S(p)$ are expressed in thousands of objects with $p \in]0;5]$.

1) Calculate the demanded number of objects for a unit price of 2 million LL.
2) Calculate the unit price for a supply of 1000 objects.
3) Determine the equilibrium price.
4) Denote by $e(p)$ the elasticity of the demand with respect to the unit price p.
   a- Show that $e(p)=\dfrac{-p\,e^{p-1}}{2+e^{p-1}}$ and calculate $e(2)$.
   b- If the unit price of 2 million LL increases by 1 %, then calculate the number of demanded objects.

Question 3. (16 points)

Choose, after justification, the correct answer for the following questions:

1) The domain of definition of the function g defined by: $g(x)=\dfrac{\ln(x+3)}{x-2}$ is:

   a- $]-3;2[\cup]2;+\infty[$

   b- $]-3;+\infty[$

   c- $]-3;2[$.

2) The solution of the equation $\ln^2(x)+5\ln(x)+4=0$ is:

   a- $S=\{e^{-1}\}$

   b- $S=\{e^{-4};e^{-1}\}$

   c- $S=\{e^{-4}\}$.

3) $\lim\limits_{x\to+\infty}\left(\dfrac{x+\ln(x)}{x}-1\right)=$

   a- -1

   b- $+\infty$

   c- 0.

4) Consider the function f defined on $]0;+\infty[$ by $f(x)=x\ln^2(x)+1$ and the expression $A=f'(e^{-1})+\ln(7-4\sqrt{3})+\ln(7+4\sqrt{3})$. The value of the expression A is:

   a- 0

   b- -1

   c- 1.

Question 4. (16 points)

A bakery offers three types of pastries: Croissants, Muffins, and Danish. Each pastry is made from a combination of three main ingredients: Flour, Sugar, and Butter. Ingredients required and the cost (**in millions of LBP**) of producing a batch of each type are given in following table:

Batch	Flour (in bags)	Sugar (in bags)	Butter (in packets)	Cost per batch
Croissants	2	1/2	1/2	4
Muffins	1	1/4	1/8	3
Danish	3	1	1/2	2.5

Assume the bakery has the following amounts of ingredients available: 25 bags of flour, 7 bags of sugar and 5 packets of butter.

Let x, y and z be respectively the number of produced batches of Croissants, Muffins, and Danish.

1) Write a system of three equations with three unknowns that described the above given.
2) By solving the previous system, deduce the number of electrical components of each type that you must use for this installation.
3) Find the total cost of the produced batches.
4) The batch selling price of each type is calculated by squaring the cost per batch then dividing it by 2. Find the achieved profit.

N.B: The question 5 is obligatory

Question 5. (32 points)

Part A: Consider the function g defined on $[0;+\infty[$ by $g(x)=1+\frac{x-3}{8}e^x$.

1) Show that $g'(x)=\frac{1}{8}(x-2)e^x$.
2) Set up the table of variations of the function g and verify that g(x) > 0 for all x on $[0;+\infty[$.

Part B: Let f be the function defined on $[0;+\infty[$ by $f(x)=x+1+\frac{x-4}{8}e^x$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to+\infty} f(x)$ and calculate $f(0)$, $f(4)$.
2) Verify that $f'(x)=g(x)$ and set up the table of variations of f.
3) Write an equation of the tangent (T) to (C) at the point with abscissa 3.
4) Plot (T) and (C).
5) The line (d) with equation $y=\frac{9}{8}x$ intersects (C) in two points with respective abscissas α and β $(\alpha<\beta)$. Darw the line (d) in the same orthonormal system as (C) and verify that $0.7<\alpha<0.9$.

Part C: (In this part take $\alpha=0.813$ and $\beta=3.919$).

A company produces objects. The total cost of production, in millions LL, is given by:

$f(x)=x+1+\frac{x-4}{8}e^x$ where x is the number, in thousands, of objects produced, with x $0\le x\le 4$.

1) Calculate the fixed cost.
2) Calculate the marginal cost of producing 2000 objects. Give an economical interpretation to the result.
3) Each produced object is sold for 1125 LL and suppose that the production is sold entirely
   - a- Show that the function of the revenue is given by $R(x)=\frac{9}{8}x$.
   - b- For what level of production will the company realize profit? Justify