Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

Entrance Exam (BC)
Mathematics Exam

14 September 2022 **Time: 2 hours**

N.B.: Choose 3 questions from the questions 1,2,3,4 (the question 5 is obligatory)

Question 1 (15 Pts)

Mr. X is going on a road trip and wants to rent a car. He needs to decide which agency to rent a car from. Agency A charges 1 million plus 5000 LBP for each kilometer he drives. Agency B charges 0.5 million plus 10000 LBP per kilometer. Agency C charges 30000 LBP for each kilometer he cuts. Denote by x the driven distance and by $f_A(x)$, $f_B(x)$ and $f_C(x)$ the costs, in LBP, for renting a car for one day from three rental agencies A, B and C respectively.

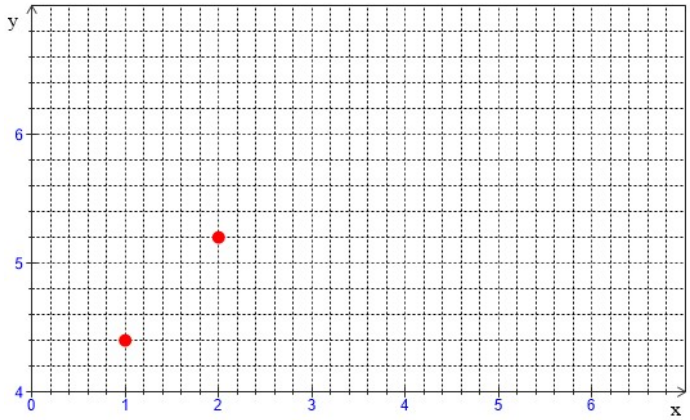
1. Verify that the cost, in thousands of LBP, for renting a car from agency A is $f_A(x) = 1000 + 5x$. Deduce the expressions of $f_B(x)$ and $f_C(x)$.
2. For what values of x, Agency A charges more than Agency B.
3. Mr. X trip is between 20 and 25 km, what agency you recommend?

Question 2 (15 Pts)

Inflation and the deterioration of the Lebanese currency value caused instability in gasoline prices in the Lebanese market. In fact, the following table shows gasoline (Benzene) tank prices (in 100,000 LBP) over 6 months starting April 2022.

Month	April	May	June	July	August	September
Rank of the month: x_i	1	2	3	4	5	6
Price of gasoline tank: y_i	4.4	5.2	6.2	6.6	6.2	6.3

1. Represent, in a similar figure as below, the scatter plot of the points (x_i, y_i) as well as the center of gravity $G(\overline{X}, \overline{Y})$. What can you notice?



2. Find the Covariance between x and y. Are x and y positively related?
3. Calculate the linear correlation coefficient and give an interpretation of its value.
4. Determine an equation of the regression line $(D_{Y/X})$ of y in terms of x; draw this line in the preceding system.

5. Suppose that the above pattern remains valid for the upcoming months in the year 2022. Find an estimation of the price of the gasoline tank during the month of November.
6. The Lebanese treasury obtains on every tank 11% tax plus 5000 LBP. It is estimated that the Lebanese market consumes 9 million tanks per month. Estimate the total income that will be brought to the treasury during the month of November.

Question 3 (15 Pts)

Choose, after justification, the correct answer for the following questions.

1. Given $A = \sqrt{\ln(\sqrt{5}-2)+\ln(\sqrt{5}+2)}$. After simplification, we get:
   a. $A = 0$
   b. $A = \sqrt{2\ln(\sqrt{5})}$
   c. A is not defined
   d. $A = \ln 2$
2. A factory produces daily a quantity x of a certain product. The total cost C of production is modelled as $C(x) = 0.8x + 1 + 4xe^{-2x},\ x \in [0,6]$ (x is expressed in hundreds of kg and $C(x)$ in millions LL). The whole production is sold at a price of 18,000 LBP per kg. Then the profit function is given by:
   a. $P(x) = -x + 1 + 4xe^{-2x}$
   b. $P(x) = 0.8x - 0.8 - 4xe^{-2x}$
   c. $P(x) = x - 1 - 4xe^{-2x}$
   d. $P(x) = 1.8x$
3. Let x and y be the two statistical variables (linearly related). The line of regression of y on x is given by: $y = 0.5x - 1.5$. One of the following statements is true:
   a. The Covariance between x and y is negative.
   b. The regression line is decreasing.
   c. The line of regression of x on y is $x = 3 + 2y$.
   d. The coefficient of correlation is positive.
4. What is $\sqrt{16\%}$?
   a. 4%
   b. 40%
   c. 0%
   d. 2.5%
5. The marginal cost M_C (derivative of cost) of production of in a certain factory is modelled as $M_C(x) = (x+1)e^{x-1}$. Then a total cost can be given as:
   a. $C(x) = e^{x-1}$
   b. $C(x) = (x-1)e^{x-1}$
   c. $C(x) = (x-1)e^{x-2}$
   d. $C(x) = xe^{x-1}$

Question 4 (15 Pts)

A box B contains balls according to the following:

- 3 red balls numbered 1, 2 and 3
- 4 blues numbered 3, 3, 4 and 4

Two balls are selected at random from the box. Consider the following events:

A "All the selected balls are red"

B "only one ball is red"

C "the balls have the number 3"

D "The sum of the numbers on both balls is 6"

E "the two balls have even number"

F "the two balls are blue with even numbers"

1. Find the probabilities of the events A, B, C, D, E and F.
2. Verify that $P(B/C)=\frac{2}{3}$.

N.B: the question 5 is obligatory

Question 5 (30 Pts)

Part A: Let f a function defined over $[1,+\infty[$ by $f(x)=\frac{2e^{-x+1}-1}{4e^{-x+1}+2}$. Designate by (C) its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$. **Important note: use the scale 1 unit = 2 cm.**

1. Complete the following table: **(Round your answer to five decimal places):**

x	0	1.5	$\ln 2+1$	2.5	3
$f(x)$					

2. Verify that $f(x)$ can be written in the form: $f(x)=\frac{2}{2+e^{x-1}}-\frac{1}{2}$.
3. Find the value of $\lim\limits_{x\to+\infty} f(x)$. Deduce an equation of the asymptote to (C)
4. Prove that the derivative of $f(x)$ can be written in the form $f'(x)=\frac{-2e^{x-1}}{\left(2+e^{x-1}\right)^2}$. **(Hint: use part 2)**
5. Find the equation of the tangent (T) to (C) at $x=0$.
6. Set up the table of variations of f.
7. Draw (T) and (C)

Part B: In this part, some economic terminologies are mentioned. Their definitions are given in the table below.

On a yearly basis, factory X produces Uninterruptible Power Supply (UPS) systems to meet the domestic demand for electricity due to energy shortage. The difference between demand $D(p)$ and supply $S(p)$ function is defined as $D(p)-S(p)=f(p)$, where f is the function given in part A. We note that, p is the unit price in millions of Lebanese pounds (LBP) with $p\in[1,4]$, $D(p)$ and $S(p)$ are expressed in hundreds of thousands of units.

1. The UPS unit is sold for 2.5 million LBP. Does the market suffer from excess in supply or excess in demand?
2. Find the market equilibrium price.
3. Suppose that the factory supplies 50,000 units per year (fixed quantity).
   a. Verify that $D(p)=\frac{2}{2+e^{p-1}}$.
   b. Find the expression of $E(p)$ the elasticity of demand with respect to the price p. Deduce the value of $E(1.5)$.
   c. Give the economical interpretation of $E(1.5)$.

Hints	**The market equilibrium price is when $D(p)=S(p)$**	**Elasticity $E(p)=-p\frac{D'(p)}{D(p)}$**