**Lebanese University**
**Faculty of Technology**

**الجامعة اللبنانيّة**
**كلية التكنولوجيا**

**Entrance Exam (BC)**
**14 September 2021** **Mathematics** **Time: 2 hours**

***N.B.: Choose 3 exercises from the exercises 1,2,3,4 (exercise 5 is obligatory)***

**Exercise 1 (16 Pts)**

Every week, the owner of a pastry shop buys 400 kg flour, 200 kg sugar and 50 kg butter at a price of 550 thousand of Lebanese Pounds (LBP.). In addition, the owner bears in his mind that sugar costs twice the price of flour. After the economic deterioration, the flour maintained the same price while the prices of sugar and butter increased by 10 and 12 times respectively. The cost of buying the same quantities of flour, sugar and butter became 4.35 million LBP. The aim is to find the initial prices of flour, sugar and butter.

1. Construct the system of equations corresponding to the above given.
2. Solve the system (you may use the calculator).
3. The owner bought, during the economic crisis, 200 kg flour, 100 kg sugar and 25 kg butter. Calculate the total sum paid.

**Exercise 2 (16 Pts)**

The table below shows the number of audience Y, in thousands, of a certain FM radio station during the years 2016-2021.

| Year | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 |
|---|---|---|---|---|---|---|
| Rank of the year: $x_i$ | 1 | 2 | 3 | 4 | 5 | 6 |
| Number of audiences: $y_i$ | 50 | 52 | 53 | 63 | 69 | 70 |

1. Represent in an orthonormal system the scatter plot of the points $(x_i, y_i)$ as well as the center of gravity $G(\overline{X}, \overline{Y})$. What can you notice?
2. Find the covariance between *x* and *y*. Is the relation that joins *x* with *y* positive or negative?
3. Calculate the linear correlation coefficient and give an interpretation of its value.
4. Determine an equation of the regression line $(D_{Y/X})$ of *y* in terms of *x* ; draw this line in the preceding system.
5. Suppose that the above pattern remains valid till 2024. Find an estimation of the number of audiences in 2022. Find the estimated percentage increase in audience.

**Exercise 3 (16 Pts)**

The staff of the FM radio station is consisting of 25 employees divided over 3 categories: Announcers, Program Directors and Technicians. 48% of the staff are Announcers and 24% are Program Directors. 75% of the Announcers are Females and all Technicians are Males. 52% of the staff are Females.

1. Copy and complete the following table.

| | **Announcers** | **Program Directors** | **Technicians** | **Total** |
|---|---|---|---|---|
| **Males** | | | | |
| **Females** | | | | |
| **Total** | | | | 25 |

2. A group formed of 5 persons are to be sent to the World Radiocommunication Conferences to represent the radio station. In how many different ways the group can be formed:
    a. Without any restrictions.
    b. Only Females are considered in the delegation.
    c. Employees of the same category are chosen.
    d. 2 Announcers, 2 Program Directors and 1 Technician.
    e. At least 4 Announcers with no Males in the delegation.
3. A person is selected at random from the radio staff. What is the probability that the selected person is either a Male Program Director or a Female Announcer?

**Exercise 4 (16 Pts)**

Find the following integrals:

1. $\int (x+1+\sqrt{x})dx$ 2. $\int \left(x+\frac{1}{x}\right)^2 dx$ 3. $\int \frac{e^x-1}{e^x-x+1}dx$

4. $\int \frac{6x}{x^2+4x-5}dx$ (Hint: Find A and B such that $\frac{6x}{x^2+4x-5}=\frac{A}{x-1}+\frac{B}{x+5}$)

**Exercise 5 (32 Pts, obligatory)**

**Part A:** Let $f$ a function defined over $[0,+\infty[$ by $f(x)=1+2e^{-x}-e^{-2x}$. Designate by $(C)$ its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$.

1. Find the value of $\lim\limits_{x\to+\infty} f(x)$. Deduce an equation of the asymptote to $(C)$.
2. Prove that the derivative of $f$ can be written in the form $f'(x)=2e^{-2x}\left(1-e^{x}\right)$.
3. Show that $(C)$ admits a horizontal tangent at a point *A* whose coordinates are to be determined.
4. Set up the table of variations of $f$.
5. Knowing that the line $(d): y=x$ intersects the curve $(C)$ at one point of abscissa $\alpha$, verify that $1.41<\alpha<1.43$.
6. Draw $(d)$ and $(C)$.

**Part B. In this part, some economic terminologies are mentioned. Note that, their definitions are given in the table below. In addition, take $\alpha\approx 1.42$.**

A factory produces candy bars to meet the demand of the local market. The monthly demand and supply functions are modelled by: $D(p)=1+2e^{-p}-e^{-2p}$ and $S(p)=p$, where $p$ is the unit price in thousands of Lebanese pounds (LBP) such that $p\in[1,6]$. $D(p)$ and $S(p)$ expressed in millions of bars.

1. The candy bar is sold for 1500 LBP.
    a. Calculate the demanded quantity. Deduce, in this case, the produced revenue.
    b. What is the amount of the oversupply?
2. Determine the market equilibrium price and the corresponding equilibrium quantity.
3. Denote by $E(p)$ the elasticity of demand with respect to the price $p$.
    a. Find the expression of $E(p)$. Deduce the value of $E(1.5)$.
    b. Give the economical interpretation of $E(1.5)$.
4. Due to the economic crisis facing Lebanon, the price of the candy bar is increased from 1500 to 6000 LBP. Find the percentage of reduction in demand.

| **Hints** | **Oversupply=Supply-Demand** | **market equilibrium price is when $D(p)=S(p)$** | **The Elasticity $E(p)=-p\frac{D'(p)}{D(p)}$** |
|---|---|---|---|

*Good luck*