Lebanese University
Faculty of Technology

الجامعة اللبنانية
كلية التكنولوجيا

Mathematics (BC), Time: 2 hours
Entrance Exam: 16 September 2020

N.B.: Choose 3 exercises from the exercises 1,2,3,4,5 (exercise 6 is obligatory)

Exercise 1 (16 Pts)

A person deposits on the first of January of the year 2010, an amount of 4,200,000LL in a bank at an annual interest rate of 6% compounded yearly. The bank charges 36,000LL every year as an annual commission. Denote by S_n the capital the person will possess on the first of January the year $2010 + n$. Let $S_0 = 4{,}200{,}000LL$.

1. Verify that $S_1 = 4416000$.
2. Show that $S_{n+1} = 1.06S_n - 36000$ for every natural number n.
3. Suppose $T_n = S_n - 600000$.
   a. Show that (T_n) is a geometric sequence whose first term T_0 and common ratio q are to be determined.
   b. Express T_n in terms of n and deduce that $S_n = 3600000 \times (1.06)^n + 600000$.
   c. On the first of January of which year will the person possess for the first time a capital that exceeds 8000000LL?
4. Calculate $C = S_0 + S_1 + \cdots + S_7$ and deduce the interest earned by this person for the first 8 years.

Exercise 2 (16 Pts)

We consider the sequence (U_n) defined by : U_1=12 and $U_{n+1} = \frac{1}{3}U_n + 5$ for all $n \geq 1$.

1. Calculate U_2 , $U_3$and U_4.
2. Let the sequence (V_n) defined, for any integer $n \geq 1$, by : $V_n = U_n - \frac{15}{2}$.
   a. Prove that the sequence (V_n) is a geometric sequence of common ration 1/3.
   b. Express V_n as a function of n.
   c. Determine the limit of the sequence (V_n) then deduce the limit of the sequence (U_n).
3. Is it possible to determine n such that: a) $U_n - \frac{15}{2} \leq 10^{-6}$ b) $U_n - \frac{15}{2} \geq 10^6$?

Exercise 3 (16 Pts)

The table below shows the price **(in thousands LL)** of $m^3$of water in a city from the year 2002 till 2006:

Year	2002	2003	2004	2005	2006
Rank of year x_i	0	1	2	3	4
Price of m^3 of water y_i	2.64	2.76	2.81	2.95	3.39

1. Represent the scatter plot of points $(x_i; y_i)$ in an orthogonal system
2. Calculate the means $\bar{x}$ and $\bar{y}$. Plot the average point G.
3. Write an equation of the regression line (D) of y in terms of x. Draw (D).
4. Estimate the price of m^3 of water in the year 2010.
5. Starting from which year will the price of m^3 of water exceeds 5000LL?

Exercise 4 (16 Pts)

Calculate each of the following integrals:

1. $\int (3x + 2)^{10} dx$
2. $\int \frac{4+\ln x}{x} dx$
3. $\int \frac{e^{2x}+x}{e^{2x}+x^2+1} dx$
4. $\int \frac{\ln x}{x^2} dx$ (using integration by parts)

Exercise 5 (16 Pts)

Employees in a school are composed of 3 categories: administrative (A), teachers (T), and simple workers (E).

- 10% of the employees are administrative and 50% are teachers.
- 80% of the administrative are males (M) and 60% of teachers are females (F), and 67.5% of the workers are males.

We choose randomly one employee from this school.

1. Give the tree diagram of this problem and precise the different probability values on it.
2. Calculate the probability that the chosen employee is an administrative woman.
3. Find the probability that an employee selected at random is a female.
4. The chosen employee is a female. Calculate the probability that this employee is from the category (E).
5. The school managers decide to increase the salary of the employees as following:
   - 100 000 LL for each employee of (A);
   - 80 000 LL for each employee of (T) and in addition, 20 000 LL for each female of (T);
   - 60 000 LL for each employee of (E) and in addition, 20 000 LL for each male of (E).

   Let X be the increase of the salaries. Demonstrate that X has 3 possible values and give the probability law of X.

Exercise 6 (32 Pts, obligatory)

Part A: Consider the function f defined over $I =]-\infty, +\infty[$ by $f(x) = \frac{1}{2}e^{2x} - 2e^x + x + 1.5$ and denote by (C) its representative curve in an orthonormal system.

1. Determine the limits of $f(x)$ at $-\infty$ and $+\infty$.
2. Show that the straight line (d) of equation $y = x + 1.5$ is an asymptote to (C) at $-\infty$. Study the relative position of (C) with respect to (d).
3. Prove that $f'(x) = (e^x - 1)^2$ and set up the table of variations of f.
4. Trace (d) and (C).
5. Show that the equation $f(x) = 2$ admits a unique solution $\alpha \in]1.2; 1.3[$.
6. Calculate the area bounded by (C), the line (d) and the lines of equations $x = 0$ and $x = \ln 4$.

Part B- Choose this Part OR Part C: (Take α=1.25) let x be the number of articles, expressed in **thousands**, produced by a factory, and $f(x)$ the total cost of production ($x \geq 0$), expressed in **millions** of LL. Each item is sold 1000LL.

7. For what value of x the cost is 2000000LL?
8. Find the revenue function $R(x)$, in millions of LL.
9. Show that the profit $P(x)$, expressed in millions of LL , is given by $P(x) = -\frac{1}{2}(e^x - 1)(e^x - 3)$.
10. For what values of x will the company realize a profit?
11. For what value of x will the company realize a maximum profit? Find this maximum.

Part C: Choose this Part OR Part B:

7. Show that f admits an inverse function $f^{-1}$and show that $(\ln 2 - 0.5; \ln 2)$ belongs to (C') the curve of f^{-1}.
8. Solve the equation $f(x)$-x=0 (you can set $e^x = y$).
9. Give the equation of the tangent (T) to (C) at the point of abscissa ln 2.
10. Study the concavity of f and show that (C) admits an inflexion point to be determined.

Good luck