Lebanese University
Faculty of Technology

الجامعة اللبنانية
كلية التكنولوجيا

Entrance Exam: Mathematics - BC

25 July 2018 **Time: 2 hours**

All questions are obligatory

Exercise I (20 points)

Part A

Consider the sequence, (u_n), defined as $u_0 = 900$, and $u_{n+1} = 0.6u_n + 200$ for every natural number n.

1) Prove that the sequence (u_n) is neither arithmetic nor geometric.
2) Consider the sequence (v_n) defined, for every natural number n, as $v_n = u_n - 500$
   a) Prove that (v_n) is a geometric sequence and determine its first term and common ratio.
   b) Prove that $u_n = 400 \times (0.6)^n + 500$
   c) Discuss the variations of the sequence (u_n).
   d) Determine the limit of the sequence (u_n).

Part B

In a certain country, two companies A and B share the communications market. The clients subscribe, the first of January, with either A or B, with a one-year contract of which they are free to choose again A or B.

Companies A and B holds 90% and 10% of the market, respectively. We estimate that, each year, 20% of the clients of A change to B, while 20% of the clients of B change to A.

Consider a population which is represented by 1000 clients in the year 2000. In this year, 900 clients are registered in A and 100 clients are registered in B.

We want to study the evolution of this population in the coming years.

1) Verify that the company A counts 740 clients in 2001.
2) Calculate the number of clients of B in 2002.
3) Denote by a_n the number of clients of A in the year $(2000 + n)$.
   a) Establish that $a_{n+1} = 0.6a_n + 200$.
   b) Using the result in **part A**, in what year the number of clients of company B exceeds 400?

Exercise II (20 points)

A factory produces plasma screens. Before being proposed to sale, each screen is tested.

If the test is positive, that the screen is well functioning, it will be offered for sale.

If the test is negative, the screen will be repaired and submitted to a new test. If the second test is positive, it will be offered for sale. Otherwise, it will be destroyed.

Assume that:

- for 70% of screens, the first test is positive.
- for 65% of repaired screens, the second test is positive.

Consider the following events:

- T: "the first test is positive";
- C: "the screen is sold".

1) We choose randomly one screen in the output of the production line. Determine the probability of the events T and C.
2) The screen is proposed to the sale. What is the probability that it comes from a first positive test?
3) The cost of production of a screen is **1000** \$ with a supplement of **50** \$ if it needs repairing. Each screen is sold at "$\boldsymbol{d}$ \$" with $\boldsymbol{d}$ is a positive number. Let X be the random variable that is equal to the algebraic gain (positive, negative or zero) realized by the factory by selling one screen.
   a) Verify that the three possible values of X are : $\boldsymbol{d-1000}$; $\boldsymbol{d-1050}$ and $\boldsymbol{-1050}$.
   b) Determine the probability distribution of X.
   c) Prove that the expected value of X is $E(X) = 0.895\boldsymbol{d} - 1015$.
   d) Starting which value of $\boldsymbol{d}$, the factory can make profits?

<u>Exercise III</u> (28 points)

Consider the function f defined on $\mathbb{R}$ as $f(x) = 3 - \frac{4}{e^{2x}+1}$

Let (C) be its representative curve in an orthonormal system (**unit 2 cm**).

1) Calculate $\lim_{x\to-\infty} f(x)$, $\lim_{x\to+\infty} f(x)$, and deduce the asymptotes to (C).
2) Prove that f is strictly increasing over $\mathbb{R}$ and set up its table of variations.
3) The curve (C) has a point of inflection W with abscissa $x = 0$. Write the equation of (T), the tangent to (C) at this point.
4) a) Calculate the abscissa of the point of intersection of (C) with the x-axis.
   b) Draw (T) and (C).
5) a) Verify that $f(x) = -1 + \frac{4e^{2x}}{e^{2x}+1}$ and deduce an antiderivative F of f.
   b) Calculate, in cm^2, the area of the region bounded by the curve (C), the x- axis, the y-axis and the line with equation $x = \ln 2$.
6) The function f has over $\mathbb{R}$ an inverse function g. Denote by (G) the representative curve of g.
   a) Specify the domain of definition of g.
   b) Show that (G) has a point of inflection, J, whose coordinates to be determined.
   c) Draw (G) in the same system as (C).
   d) Determine $g(x)$ in terms of x.

<u>Exercise IV</u> (12 points)

Calculate the following integrals:

1) $\int \frac{dx}{x^2-5x+6}$ 2) $\int \frac{2x-3}{\sqrt{x^2-3x+2}}dx$ 3) $\int_{-1}^{2}|x-1|\,dx$ 4) $\int xe^{-x}dx$