LEBANESE UNIVERSITY
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الجامعة اللبنانية
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Entrance Exam: September 11, 2014
Mathematics : ABC Duration: 2 H

Choose 3 from the exercises 1,2, 3, and 4

Exercise 1. (12 Pts)

Consider two urns U and V :

U contains **five** balls: **three** balls numbered 0 and **two** balls numbered 1.

V contains **five** balls numbered 1 to 5.

A - One ball is drawn randomly from each urn. Designate by X the random variable that is equal to the product of the two numbers that are marked on the two drawn balls.

1) Prove that $P(X = 0)$ is equal to 3/5.

2) Determine the probability distribution of X.

B – In this part, the 10 balls that were in urns U and V are all placed in one urn W.
Two balls are drawn, simultaneously and randomly, from this urn W.

1) What is the number of possible draws of these 2 balls?

2) Let q designate the product of the two numbers that are marked on the two drawn balls.

a) Show that the probability $P(q = 0)$ is equal to 8/15.

b) Calculate the probability $P(q < 4)$.

Exercise 2. (12 Pts)

The following table shows the salary change (y) of a worker expressed in hundreds of thousands of Lebanese pounds since 2010:

Date	1/7//2011	1/7/2012	1/7/2013	1/7/2014	1/7/2015	1/7/2016	1/7/2017
Rank : x_i	1	2	3	4	5	6	7
Salary : y_i	6,67	6,83	7,19	7,61	8,03	8,27	8,44

1. Represent graphically by scatter diagram the data in the above table in an orthogonal coordinate system (1 cm represents 1 row x-axis and 5 cm represent 100,000 LL y-axis. Start y-axis from 6).
2. Give an equation for the regression line of y with respect x by the least squares method (round coefficients to 10^{-2}). Draw this obtained line in the above graph.
3. The shape of the scatter diagram suggests a modification of the salary trend from July 2014. For $x \geq 4$, we choose to adjust the regression line by the curve (C) of equation: $y = a \ln(x-3) + b$ where a and b are two real numbers.

a. Determine the values of a and b such that the curve (C) passes through the points with coordinates (4, 7.61) and (7, 8.44) (round the real numbers a and b to 10^{-2}).

b. Draw the curve (C) on the same graph of question 1.

4. Sami is an employee having (y) salary. He wants to estimate (y) on July 1, 2019. What is the most favorable model?

Exercise 3. (12 Pts)

Let (u_n) a numeric sequence defined by: $u_1 = 12$ and $u_{n+1} = \frac{1}{3}u_n + 5$ for any integer $n \geq 1$

1. Compute u_2, u_3 and u_4.
2. Let the sequence (v_n), defined for $n \geq 1$, by : $v_n = u_n - 15/2$
   a. Prove that (v_n) is a geometric sequence with a common ratio 1/3
   b. Write then v_n in function of n
   c. Compute the limit of (v_n), and then deduce the limit of (u_n).
3. Is it possible to determine n to have: a. $u_n - 15/2 \leq 10^{-6}$? b. $u_n - 15/2 \geq 10^{6}$?

Exercise 4. (12 Pts)

An apartment is valued at 100 ML (Million Lebanese Pounds) on 1/1/2014. The annual increase in the apartment value is 4% (supposed constant).

1. What would the value of the apartment be on 1/1/2015 ? and on 1/1/2016 ?
2. Let $U_0 = 100$ and U_n the apartment value for the year (2014+n).
   a) Give the relation between U_{n+1} and U_n and deduce that (U_n) is a geometric sequence whose common ratio is to be determined.
   b) Express U_n in term of n.
   c) After how many years would the value of the apartment be doubled?
3. This apartment is rented 0.3ML / month. The rent is fixed. The owner has the option to sell the apartment for 100ML and to put this capital in the bank at an interest rate of 6.5%. Which option is more profitable after: a) 5 years ? b) 10 years ?

Exercices 5 and 6 are obligatory

Exercise 5. (24 Pts)

Consider the function f defined on $]0; +\infty[$ by : $f(x) = 2x + \ln[2x/(2x+1)]$

Let (C) be the curve representing f in an orthonormal system $(O ; \vec{i}, \vec{j})$.

Part A :

1) a- Calculate $\lim_{x\to+\infty} f(x)$ and deduce the asymptote line (D) to (C).

b- Verify that $2x/(2x+1) < 1$ and deduce that (C) is below (D).

2) Calculate $\lim_{x\to 0} f(x)$ and deduce the asymptote to (C).

3) Compute $f'(x)$ and set up the table of variations of f.

4) Draw (D) and (C).

Part B : A study of the market showed that:

- The quantity of objects produced by a factory is modeled by the function f.
- The quantity of objects demanded from this factory is modeled by the function $g(x) = 2x + 1$.
- $f(x)$ is the quantity of objects produced by this factory expressed in thousands. x *is* expressed in weeks ($1 \le x \le 10$), and $g(x)$ is the quantity of demanded objects in thousands.

1) The «demand is satisfied on the date x» if $f(x) \ge g(x)$. Show that the demand is never satisfied.

2) Assume that the total number of objects, in thousands, whose demand is not satisfied between two dates n and m is given by $\int_n^m [g(x) - f(x)]dx$.

a- Calculate the antiderivative H of the function $g(x) - f(x)$.

b- Compute the total number of objects whose demand is not satisfied : from n=1 to m=5.

Exercise 6. (10 Pts)

Consider a function f defined and derivable on the interval [-5, 6] which has an antiderivative over this interval. The table on the right gives f variations.

x	−5		−3		2		4		6
$f(x)$	3	↘	1	↗	4	↘	−2	↗	0

In each of the following questions, choose and justify the correct answer:

1 – For 2 real numbers a and b where $2 < a < b < 4$ then:

a) $f(a) > f(b)$ b) $f(a) < f(b)$ c) f(a) and f(b) cannot be compared

2 - The number of solutions of the equation $f(x) = 1$ is: a) 1 b) 2 c) 3

3 - a) $\int_4^5 f(x)dx < 0$ b) $\int_4^5 f(x)dx > 0$ c) The sign of $\int_4^5 f(x)dx$ is undetermined

4 - Let g the function defined on [-5, 6] by $g(x) = -1 + x/2$. The equation $f(x) = g(x)$:

a) has no solution b) has one solution c) the number of solutions cannot be determined