Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

Entrance Exam - Engineering

Math Exam

September 09, 2025 **(Grade/50)** **Time: 2 hours**

N.B: Choose three questions from the questions 1, 2, 3 and 4.
The question 5 is obligatory.

Question 1. (10 points)

In the table below, only one answer among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Answers A	Answers B	Answers C
1	Given: $a = 8 + \ln\left(\frac{1}{e^2}\right) - 3\ln(e^2)$ and $b = \ln\left(\sqrt{9+e} - 3\right) + \ln\left(\sqrt{9+e} + 3\right)$ Then:	$a = b$	$a > b$	$a < b$
2	Let $X = \lim\limits_{x \to 0}(2 + x\ln x)$ and $Y = e^{3\ln X}$ Then:	$Y = 6$	$Y = 8$	$Y = 0$
3	The domain of definition of the function $f(x) = \ln(-x^2 + 1)$ is:	$]-1; 1[$	$]-1; +\infty[$	$]-\infty; -1[\cup]-1; +\infty[$
4	$\displaystyle\int_{\sqrt{e-1}}^{\sqrt{e^2-1}} \frac{x}{(x^2+1)\ln(x^2+1)}\,dx =$	$2\ln 2$	$\ln\sqrt{2}$	0.3

Question 2. (10 points)

In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the points: $A\,(4\,;\,2\,;\,0)$, $B\,(2\,;\,3\,;\,1)$ and $C\,(2\,;\,2\,;\,2)$.

1) Prove that triangle ABC is right at B.

2) Show that an equation of the plane (P) determined by the three points A, B and C is: $x + y + z - 6 = 0$.

3) Let (Q) be the plane passing through A and perpendicular to (AB).
a- Determine an equation of (Q).
b- Denote by (D) the line of intersection of (P) and (Q), show that (D) is parallel to (BC).

4) Let $H(5; 3; 1)$ be a point in (Q).
a- Show that A is the orthogonal projection of H on (P).
b- Calculate the volume of the tetrahedron HABC.

Question 3. (10 points)

In the complex plane referred to a direct orthonormal system$(O\,; \vec{u}, \vec{v})$, consider the points A, B, C, M and M' of affixes 1, 2, 3, z and z' respectively such that $z' = \frac{\bar{z}-3}{\bar{z}-2}$ where $z \neq 2$ and M distinct from B.

1) Determine the set of points M if $|z'| = 1$.

2) a- Verify that $z' - 1 = \frac{-1}{\bar{z}-2}$.
b- Deduce that $AM' \times BM = 1$.

3) Suppose that $z = 2 + e^{i\theta}$, where $\theta \in]-\pi; \pi[$. Write $z' - 1$ in the exponential form.

4) Let $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers.

a- Verify that $x' = \frac{x^2+y^2-5x+6}{(x-2)^2+y^2}$ and $y' = \frac{-y}{(x-2)^2+y^2}$.

b- Determine the set of points M such that z' is pure imaginary.

c- Determine the set of points M' such that M describes the line of equation $x = 2$.

Question 4. (10 points)

An urn U contains ten balls:

- five white balls numbered 1, 2, 3, 4, 5
- three black balls numbered 6, 7 ,8
- two green balls numbered 9, 10

Part A: A player selects randomly and simultaneously two balls from the urn U. Consider the following events:

A: « The two selected balls hold odd numbers»,

B: « The two selected balls have the same color»,

C: « The two selected balls hold odd numbers and have the same color»,

D: « The two selected balls hold odd numbers and have different colors».

1) Calculate the probability $P(A)$ and verify that $P(B) = \frac{14}{45}$.

2) a- Calculate $P(C)$.

b- Are the events A and B independent? Justify.

3) Verify that $P(D) = \frac{7}{45}$.

4) Knowing that the player has selected two balls with different colors, what is the probability that these two balls hold odd numbers?

Part B: In this part, the player selects randomly, successively and with replacement, two balls from the urn U. The player scores **+1** point for each white ball selected, **–1** point for each black ball selected and **0** points for each green ball selected. Calculate the probability that the sum of scored points is equal to zero.

N.B: The question 5 is obligatory.

Question 5. (20 points)

Part A: Consider the function g defined over $\mathbb{R}$, by: $g(x) = -2 + xe^{-x+1}$.

1) Calculate $\lim\limits_{x\to-\infty} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.

2) Calculate $g'(x)$ and set up the table of variations of g.

3) Deduce the sign of $g(x)$.

Part B: Consider the function f defined over $\mathbb{R}$, by: $f(x) = -2x + 3 - xe^{-x+1}$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$. Let (d) the line of equation $y = -2x + 3$.

1) Determine $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.

2) a- Determine $\lim\limits_{x\to+\infty} [f(x) - (-2x + 3)]$ and deduce that the line (d) is an asymptote to (C).

b- Study, according to the values of x, the relative position between (C) and (d).

3) Verify that $f'(x) = g(x) - e^{-x+1}$, then set up the table of variations of f.

4) Show that the curve (C) admits an inflexion point I whose coordinates are to be determined.

5) Calculate $f(0)$ and $f(1)$, then draw (C) and (d).

6) a- Show that $\int xe^{-x+1}\,dx = -(x + 1)e^{-x+1} + K$.

b- Calculate the area of the region bounded by (C), (d) and the two lines with equations $x = 0$ and $x = 2$.