Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

Entrance Exam (TSE)
Mathematics Exam

September 25, 2023 **Time: 2 hours**

N.B.: Choose three questions from the questions 1, 2, 3 and 4 (the question 5 is obligatory)

Question 1. (10 points)

In the table below, only one answer among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Answers		
		a	**b**	**c**
1	The solution of the equation: $C_{3n}^2 = A_n^2 + 30$ is	2	3	4
2	$\lim_{x \to -\infty} \frac{\ln(e-3x)}{\ln(1-x)} =$	$+\infty$	e	1
3	The equation $\ln^2 x - 2\ln\left(\frac{e}{x}\right) + 3$ admit	One solution	two solutions	No solution
4	The domain of definition of the function *f* given by: $f(x) = \ln(1 - \ln(x))$ is:	$]-\infty, e[$	$]0, e[$	$]0, +\infty[$
5	If $2p(A) = p(B) = \frac{5}{13}$ and $p\left(A/B\right) = \frac{2}{5}$, then $p(\overline{A} \cap \overline{B}) =$	$\frac{10}{26}$	$\frac{11}{26}$	$\frac{15}{26}$

Question 2. (10 points)

We have a perfect dice whose faces are numbered from 1 to 6 and an urn U containing **two balls** carrying strictly **positive** numbers and **three balls** carrying strictly **negative** numbers.
The dice is rolled:

- If the number appearing is 1 or 2, two balls are drawn randomly and simultaneously from the urn U.
- If the number appearing is 3, 4, 5 or 6, three balls are drawn randomly and simultaneously from the urn U.

We consider the following events:

A: « The number that appeared on the dice is 1 or 2 ».
B: « The number that appeared on the dice is 3, 4, 5 or 6 ».
C: « The **product** of the numbers carried by the balls drawn from the urn U is **negative**».

1) Show that $p(A) = \frac{1}{3}$.
2) a- Show that $p\left(C/A\right) = 0.6$ and deduce $p(C \cap A)$.

   b- Show that $p\left(C/B\right) = 0.4$ and deduce $p(C \cap B)$.

   c- Calculate $p(C)$.
3) **The product** of the numbers carried by the balls drawn from the urn U is **positive**, what is the probability that the number appearing on the dice is 1 or 2?

Question 3. (10 points)

Suppose that you are working on designing of an electrical network for an industrial installation. You have three types of electrical components: transformers, circuit breakers, and capacitors. You need to install a total of 30 electrical components in the network. Transformers cost 500 $ each, circuit breakers cost 100 $ each, and capacitors cost 50 $ each. Moreover, the total number of capacitors and circuits breakers is equal to the number of transformers. You have a total budget of 8500 $ for the project.

Let x, y and z be respectively the numbers of transformers, circuit breakers, and capacitors.

1) Write a system of three equations with three unknowns that described the above given.
2) By solving the previous system, deduce the number of electrical components of each type that you must use for this installation.

Question 4. (10 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points E, A, and M of affixes 2i, -i, z respectively, and let M' be the point of affix z' so that $z' = 2i - \frac{2}{z}$ $(z \neq 0)$.

1) a- Show that $z\left(z' - 2i\right) = -2$.

   b- Calculate $\arg(z) + \arg\left(z' - 2i\right)$.

2) a- Verify $z' = \frac{2i(z+i)}{z}$, and deduce that $OM' = \frac{2AM}{OM}$.

   b- As M moves on the perpendicular bisector of $[OA]$, prove that M' moves on a circle (C) whose center and radius are to be determined.

3) Suppose that $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers.

   a- Show that $x' = \frac{-2x}{x^2+y^2}$ and $y' = 2 + \frac{2y}{x^2+y^2}$.

   b- If $x = y$, show that the lines (OM) and (EM') are perpendicular.

N.B.: The question 5 is obligatory

Question 5. (20 points)

Part A:

Let g be the function defined over $\mathbb{R}$ by: $g(x) = -1 - (x+1)e^{-x}$.

1) Calculate $\lim\limits_{x\to-\infty} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.
2) Calculate $g'(x)$ and set up the table of variations of g.
3) Show that the equation $g(x) = 0$ has a unique solution α and verify that $-1.3 < \alpha < -1.2$
4) Discuss, according to the values of x, the sign of $g(x)$ over $\mathbb{R}$.

Part B:

Consider the function f defined over $\mathbb{R}$ by: $f(x) = -x + (x+2)e^{-x}$ and designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Verify that $\lim\limits_{x\to-\infty} f(x) = -\infty$ and calculate $f(-2.5)$ to the nearest 10^{-2}.
2) a- Determine $\lim\limits_{x\to+\infty} f(x)$ and prove that the straight line (d) of equation $y = -x$ is an asymptote to (C).

   b- Study the relative position between (C) and (d).
3) Verify that $f'(x) = g(x)$ and set up the table of variations of f.
4) Show that the curve (C) admits an inflection point I whose coordinates should be determined.

5) Verify that $f(\alpha)=\dfrac{-\alpha^2-2\alpha-2}{\alpha+1}$.

6) Draw (d) and (C)(take $\alpha=-1,28$).

<u>Part C</u>:

Consider the function h defined by $h(x)=\ln\left[f(x)-2\right]$, and designate by (H) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Find the domain of definition of h.
2) Find the abscissa of point A on (H) where the tangent to (H) at A is parallel to the x-axis.