Lebanese University

Faculty of Technology



#### Entrance Exam (Engineering) Mathematics Exam

September 15, 2022

Time: 2 hours

N.B.: The questions 1, 2 and 3 are obligatory (choose one of the two questions 4 or 5)

## Question 1. (8 points)

In the table below, only one answer among the proposed answers to each question is correct. Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Answers		
		a	b	c
1	The solution of the equation: $\ln(3x+4) + \ln(-2x+6) = \ln 5 + \ln 4$ is	$\frac{-1}{3}$	2	$\frac{-1}{3}$ or 2
2	$\lim_{x \to +\infty} \left( x - \frac{e^{2x}}{x} \right) =$	$+\infty$	-∞	1
3	The derivative of the function f given by: $f(x) = \frac{e^{x} - 2x}{e^{x}}$	$\frac{2-2e^{x}}{\left(e^{x}\right)^{2}}$	$\frac{2e^{x}-2}{\left(e^{x}\right)^{2}}$	$\frac{2x-2}{e^x}$
4	The domain of definition of the function h given by: $h(x) = \ln\left(\frac{e^{x} + 1}{e^{x} - 1}\right)$ is	$]{-\infty,0[}$	$]-\infty,+\infty[$	]0,+∞[

## Question 2. (12 points)

A clothing stores sells casual and formal dresses only. The customers of this store are surveyed and the results are as follows:

- 30 % of these customers bought each a casual dress.
- Out of those who bought a casual dress, 60 % bought a formal dress.
- 28 % of the customers bought each a formal dress.

A customer of the store is interviewed at random. Conisder the following events:

C : « the interviewed customer bought a casual dress »;

F : « the interviewed customer bought a formal dress ».

## Part A :

- 1) Calculate the probability  $P(C \cap F)$  and verify that  $P(C \cap \overline{F}) = 0.12$ .
- 2) Prove that  $P(\overline{C} \cap \overline{F}) = 0.6$ .
- 3) Calculate the probability that the customer bought at least one dress.
- 4) Knowing that the customer didn't buy a formal dress, calculate the probability that she bought a casual dress.

#### <u> Part B</u> :

On Sunday, 30 customers visited this store. We choose, simultaneously and at random, three of these customers. Consider the following events:

A : « among the three chosen customers exactly one bought a casual dress ».

B: « among the three chosen customers at most two bought a casual dress ».

1) Verif that 
$$P(A) = \frac{27}{58}$$
.

2) Calculate P(B).

# Question 3. (25 points)

**<u>Part A</u>**: Let g be the function defined over  $]0, +\infty[$  by:  $g(x) = x^3 - 1 + 2 \ln x$ .

- 1) Determine  $\lim_{x\to 0} g(x)$  and  $\lim_{x\to +\infty} g(x)$ .
- 2) Calculate g'(x) and set up the table of variations of g.
- 3) Calculate g(1), then deduce the sign of g(x) according to the values of x.

**<u>Part B</u>**: Consider the function f defined over  $]0, +\infty[$  by:  $f(x) = x - \frac{\ln x}{x^2}$  and designate by (C) the

representative curve of f in an orthonormal system  $(O; \vec{i}, \vec{j})$ . Let (d) be the line with equation y = x.

- 1) Determine  $\lim_{x \to \infty} f(x)$  and deduce an asymptote to (C).
- 2) Study according to values of x, the relative position of (C) with respect to (d).
- 3) Determine  $\lim_{x \to +\infty} f(x)$  and show that the line (d) is an asymptote to (C).
- 4) a-Verify that  $f'(x) = \frac{g(x)}{x^3}$  and set up the table of variations of f.
- 5) Determine the point E of (C) where the tangent (T) to (C) is parallel to (d).
- 6) Draw (d) and (C).

#### N.B.: Choose one of the two questions 4 or 5

#### Question 4. (5 points)

A movie theater has a seating capacity of 350. The theater charges \$4.50 for children, \$7.50 for students, and \$12.50 for adults. There are half as many adults as there are students. The total ticket sales was \$2415. Let x, y and z be respectively the numbers of children, students and adults.

- Write a system of three equations with three unknowns that described the above given.
- 2) By solving the previous system, deduce the number of children, students and adults attended.

#### Question 5. (5 points)

In the complex plane referred to a direct orthonormal system (O; u, v), consider the points A, B, and M of

affixes -i, -2, z respectively, and let M' be the point of affix z' so that  $z' = \frac{1-iz}{z+2}$   $(z \neq -2)$ .

1) a- Find the algebraic form of the complex number (z'+i)(z+2).

b-Give a geometric interpretation to |z'+i| and |z+2| then deduce that  $AM' \times BM = \sqrt{5}$ .

c- As M moves on the circle with center B and radius 1, show that M' moves on a circle whose center and radius are to be determined.

- 2) Suppose that z = -2 + iy with y a nonzero real number.
  - a- Find in terms of y the algebraic form of z '.
  - b- Determine the point M for which z' is real.