Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

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Entrance Exam (Engineering)
Mathematics Exam

September 15, 2021 **Time: 2 hours**

N.B.: The questions 1, 2 and 3 are obligatory (choose one of the two questions 4 or 5)

Question 1. (8 points)

In the table below, only one answer among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Answers		
		a	**b**	**c**
1	For all real numbers x, we have: $\ln\left(1+e^{-x}\right)-\ln\left(1+e^{x}\right)=$	0	-x	x
2	$\lim\limits_{x\to 1}\dfrac{e^{x}-e}{x-1}=$	$+\infty$	1	e
3	Let f be the function given by: $f(x)=\dfrac{\ln\left(x^2+1\right)}{\ln(x)}$ The domain of definition of f is:	$]-\infty;+\infty[$	$]0;1[\cup]1;+\infty[$	$]-\infty;0[\cup]0;+\infty[$
4	If A and B are two events such that $P(A)=0.4$, $P(B)=0.8$ and $P\left(B/A\right)=0.6$ then $P(A\cup B)$ equals to	0.24	0.96	0.48

Question 2. (9 points)

Consider two urns U and V. The urn U contains two balls numbered 1 and 2 and the urn V contains four balls numbered 1, 2, 3 and 4.

1) One of the two urns U and V is randomly chosen, after which a ball is randomly selected from this urn. Consider the following events:
   - U: « the chosen urn is U ».
   - N: « the selected ball is numbered 1 ».

   a-Calculate the probabilities $P(N/U)$ and $P(N\cap U)$.

   b- Show that $P(N)=\dfrac{3}{8}$ and deduce $P(U/N)$.

2) In this part, the six balls from the two urns U and V are placed in one urn W. Two balls are selected randomly and simultaneously from the urn W. Consider the following events:
   - E: « the two selected balls carry the same number ».
   - F: « the sum of numbers carried by the two selected balls is odd ».

   a- Verify that $P(E)=\dfrac{2}{15}$.

   b- Calculate $P(F)$ and $P\left(F/\overline{E}\right)$.

Question 3. (25 points)

Part A: Let g be the function defined over $\mathbb{R}$ by: $g(x) = (1-x)e^{-x} + 1$.

1) a- Find $\lim\limits_{x \to +\infty} g(x)$.

   b- Find $\lim\limits_{x \to -\infty} g(x)$.

2) a- Calculate $g'(x)$ and set up the table of variations of g.

   b- Deduce that $g(x) > 0$ for all x in $\mathbb{R}$.

Part B: We consider the function f defined over $\mathbb{R}$ by: $f(x) = xe^{-x} + x$ and designate by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Determine $\lim\limits_{x \to +\infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$.
2) Show that the line (d) with equation $y = x$ is an asymptote to (C).
3) Study the relative position of (C) with respect to (d).
4) a- Verify that $f'(x) = g(x)$ and set up the table of variations of f.

   b- Show that $I\left(2, 2 + \frac{2}{e^2}\right)$ is an inflection point of (C).

   c- Determine the point E of (C) where the tangent (T) to (C) is parallel to (d).

   d- Draw (d), (T) and (C).
5) Calculate A, the area of the region bounded by (C), (d) and the two vertical lines with equations $x = -1$ and $x = 1$.

N.B.: Choose one of the two questions 4 or 5

Question 4. (8 points)

A factory manufactures three models of computers videos cards every day: model C1, model C2 and model C3. Electronic chips of the types: P1, P2 and P3 are installed inside each model with the following distribution:

Chip \ Model	C1	C2	C3
P1	5	2	7
P2	3	8	6
P3	3	4	5

In a certain day, we used: 200 chips P1, 240 chips P2 and 170 chips P3.
Let x, y and z be respectively the numbers of the manufactured cards C1, C2 and C3.

1) Write a system of three equations with three unknowns that described the above given.
2) By solving the previous system, deduce the number of cards manufactures from each model.

Question 5. (8 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A, B, and M of affixes -1, 4, z respectively, and let M' be the point of affix z' so that $z' = \frac{z-4}{z+1}$ $(z \neq -1)$.

1) In the case where $z = 1 + i$, write z' in its algebraic form, and give its exponential form.
2) Determine the values of z for which z' = z.
3) a- Give a geometric interpretation of $|z+1|$ and $|z-4|$.

   b- Find, when $|z'| = 1$, the line on which the point M moves.