Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

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Entrance Exam (Engineering)
Mathematics Exam

Septembre 17, 2020 **Time: 2 hours**

N.B.: The questions 1, 2, 3 are obligatory

Exercise 1. (28 points)

Part A

Let g be the function defined over $]0;+\infty[$ by: $g(x) = x + \ln(x)$.

1) Determine $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$.

2) Calculate $g'(x)$ and set up the table of variations of g.

3) a- Prove that the equation $g(x) = 0$ has a unique solution α, and verify that $0.5 < \alpha < 0.6$.

   b- Discuss, according to the values of x, the sign of $g(x)$.

Part B

Let f be the function defined over $]0;+\infty[$: $f(x) = x(2\ln x + x - 2)$ and designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to 0} f(x)$, $\lim\limits_{x\to +\infty} f(x)$ and calculate f (e).

2) Show that $f(\alpha) = -\alpha(\alpha + 2)$.

3) Verify that $f'(x) = 2g(x)$ and set up the table of variations of f(x).

4) Draw (C). (Take $\alpha = 0{,}55$).

5) Use integration by parts to calculate $\int_{0,5}^{1} x\ln x\,dx$ and deduce the area of the region bounded by the curve (C), the axis of abscissas and the two lines with equations x=0.5 and x =1.

Exercise 2. (14 points)

A box V contains cards such that:

- 20% of the cards are blue and the other cards are red;
- 40% of the blue cards have odd numbers;
- 32% of the total cards carry odd numbers.

1) A card is randomly selected from box V.

   Consider the following events:

   - B : «select a blue card »
   - R : «select a red card »
   - O: «select a card carrying an odd number ».

   a- Calculate the probabilities $p(O \cap B)$ and verify that $p(O \cap R) = 0.24$.

   b- Deduce $p(O/R)$.

   c- The selected card does not carry an odd number, what is the probability that it is red?

2) In this part, suppose that the number of cards in box V is 50. Three cards are randomly and simultaneously selected from V.

   Consider the following events:

   - M: «among the three selected cards, exactly two carry odd numbers»
   - N: «the three selected cards are blue»
   - L: « among the three selected cards, exactly two carry odd numbers and one is blue».

Calculate the probability $p(M)$; $p(N/M)$ and $p(L)$.

Exercise 3. (10.5 points)

Rami withdraws from the ATM of his bank a sum of 725 \$. The ATM gave him 45 bills, including:

- Bills of 5\$
- Bills of 10\$
- Bills of 20\$

After he left the bank, Rami went to a store. After passing through the cashier, he had half number of 10 \$ bills, half bills of 20 \$, same number of 5 \$ bills and a total sum of 375 \$.

1) Write a system of three equations with three unknowns that translates the text above.
2) By solving the previous system, calculate the number of bills of 5\$, 10\$ and 20\$ that remain with Rami after leaving the store.

NB: Choose one of the two questions 4 or 5

Exercise 4. (17.5 points)

In the space of an orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the plane (P) with equation:

$x + y + z - 1 = 0$, and the two lines (d) and (d') defined as:

$(d): \begin{cases} x = m + 1 \\ y = 2m + 1 \\ z = 2m + 2 \end{cases}$ and $(d'): \begin{cases} x = 2t \\ y = 5t - 3 \\ z = 4t \end{cases}$ (m and t are real parameters)

1) Find the coordinates of A, the common point between (d) and (P).
2) Verify that A is on line (d'), and that (d') is contained in plane (P).
3) a- Write an equation of plane (Q) determined by the lines (d) and (d').

   b- Show that the two planes (P) and (Q) are perpendicular.
4) Let $B(1;1;2)$ be a point on (d).

   Calculate the distance from point B to line (d')

Exercise 5. (17.5 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A, B, M and M′ with respective affixes 2, -i, z and z' so that $z' = \dfrac{iz - 1}{z - i}$, $(z \neq i)$.

1) Find the coordinates of M when $z' = 1 + 2i$.
2) Give a geometric interpretation for $|z - 1|$, $|iz - 1|$ and determine the set of points M such that

   $|z - 1| = |iz - 1|$.
3) Let $z = x + iy$ and $z' = x' + iy'$
   a- Calculate x' and y' in terms of x and y.
   b- Show that if z' is pure imaginary, then M moves on a straight line whose equation is to be determined.
   c- Show that if z is real, then M' moves on a straight line whose equation is to be determined.