Lebanese University
Faculty of Technology

الجامعة اللبنانيّة
كلية التكنولوجيا

Entrance Exam (Engineering)
Mathematics Exam

July 24, 2018 **Time: 2 hours**

N.B.: All question are obligatory.

Exercise 1 (10 points)

In the space of an orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the plane (P) with equation :

$x+y+z-1=0$, and the line (d) with parametric equations: $\begin{cases} x=-m-1 \\ y=m+5 \quad (m\in\mathbb{R}), \\ z=3m+9 \end{cases}$

and H (1, 1, -1) be a point of (P).

1) Determine the coordinates of A, the common point between (d) and (P).
2) Let (Δ) be the line passing through H and perpendicular to the plane (P).
   a- Write a system of parametric equations of (Δ).
   b- Verify that E (2, 2, 0) is the intersection point between (Δ) and (d).
3) Let (Q) be the plane passing through O and the point F (2, 1, 0), and perpendicular to (P).
   a- Write an equation of the plane (Q).
   b- Let $M(x,y,z)$ be a variable point on (Q). Prove that the volume of the tetrahedron MEAH is constant.

Exercise 2. (10 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B, M and M′ with respective affixes -2, i, z and z' so that $z'=\dfrac{z+2}{z-i}$

1) In this part only, assume that $z=\sqrt{2}e^{i\frac{3\pi}{4}}$.
   a- Write the complex number $-1-i$ in exponential form.
   b- Deduce that $(z')^{40}$ is a real number.
2) Let $z=x+iy$ and $z'=x'+iy'$ where x, y, x′ and y′ are real numbers.
   a- Calculate x' and y' in terms of x and y.
   b- Express the scalar product $\overrightarrow{AM}.\overrightarrow{BM}$ in terms of x and y.
   c- Deduce that if z′ is pure imaginary, then the two lines (AM) and (BM) are perpendicular.
3) a- Verify that $(z'-1)(z-i)=2+i$.
   b- Deduce that if M moves on the circle (C) with center B and radius $\sqrt{5}$, then M′ moves on a circle (C′) with center and radius to be determined.

Exercise 3. (10 points)

Consider an urn U containing three dice:

- **Two** red dice where the faces of each of them are numbered from 1 to 6.
- **One** black die where two of its faces are numbered 6 and the four others are numbered 1.

A player selects randomly and simultaneously two dice from the urn, then he rolls them only once. Consider the following events:

A: «The two dice selected are red».

$\overline{A}$: «The two dice selected are one red and one black».

L: «Out of the two dice, only one shows the number 6».

1) Calculate the probability $p(A)$.
2) a- Verify that $p(L / A) = \frac{5}{18}$ and calculate $p(A \cap L)$.

   b- Calculate $p(\overline{A} \cap L)$ verify that $p(L) = \frac{19}{54}$.
3) Knowing that only one of the two dice shows the number 6, calculate the probability that the two dice selected are red.
4) Calculate the probability that at least one die shows the number 6.

Exercise 4. (20 points)

Consider the function f defined on $\mathbb{R}$ by $f(x) = (x+1)^2 e^{-x}$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-2)$.

   b- Determine $\lim\limits_{x \to +\infty} f(x)$ and deduce an asymptote to (C).
2) Show that $f'(x) = (1 - x^2)e^{-x}$ and setup the table of variation of f.
3) The line (d) with equation $y = x$ intersects (C) at a point with abscissa α.
   Verify that $1{,}4 < \alpha < 1{,}5$.
4) Draw (d) and (C).
5) Let F be the function defined on $\mathbb{R}$ by $F(x) = (px^2 + qx + r)e^{-x}$.

   a- Calculate p, q and r so that F is an antiderivative of f.

   b- Calculate the area of the region bounded by the curve (C), the axis of abscissas and the two lines with equations $x = 0$ and $x = 1$.
6) Prove that the function f has, on $[1; +\infty[$ an inverse function f^{-1}. Determine the domain of definition of f^{-1} and draw its representative curve in the same system as (C).