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الجامعة اللبنانية
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Entrance Exam: September 10, 2014
Mathematics : CE - CCNE **Duration: 2 H**

N.B.: All questions are obligatory

Exercise 1. (12 Pts)

The 20 employees in a factory are distributed into two departments as shown in the table below:

	Technical Department	Administrative Department
Women	3	5
Men	10	2

1) The manager of this factory wants to offer a gift to one of the employees. To do this, he chooses randomly an employee of this factory.

Consider the following events:

W : « the chosen employee is a woman ».

M : « the chosen employee is a man ».

T : « the chosen employee is from the technical department ».

A : « the chosen employee is from the administrative department ».

a- Calculate the following probabilities:

$P(W/T)$, $P(W/A)$, $P(W \cap T)$ and $P(W)$.

b- Knowing that the chosen employee is a man, what is the probability that he is from the technical department ?

2) On a different occasion, the factory manager chooses **two** employees randomly and simultaneously from the technical department and also chooses **one** employee randomly from the administrative department.

Let X the random variable that is equal to the number of women chosen.

a- Verify that $P(X=1)=95/182$.

b- Determine the probability distribution of X.

Exercise 2. (12 Pts)

Let f be the function defined on IR by $f(x) = (x-1)e^x + 1$ and (C) its representative curve in an orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j}\ )$.

1) a- Calculate $\lim\limits_{x \to -\infty} f(x)$ and deduce the asymptote (d) of (C).

b- Study, according to the values of x, the relative positions of (C) and (d).

c- Calculate $\lim\limits_{x \to +\infty} f(x)$ and find $f(2)$ in decimal form.

2) Calculate $f'(x)$ and set up the table of variations of f.

3) Prove that the curve (C) has a point of inflection W whose coordinates are to be determined.

4) Draw (d) and (C).

5) Calculate the area of the region bounded by (C), the axis of abscissas and the two lines of equations $x=0$ and $x=1$.

Exercise 3. (10 Pts)

The solutions of the differential equation $y' = ay$ where $a \in IR$ are the fonctions g defined on IR by : $g(x) = Ke^{ax}$ ($K \in IR$). We want to determine the solutions of the differential equation (E) $y' = ay + b$ where $a \in IR^*$ and $b \in IR$.

1. Prove that the function u defined on IR by $u(x) = \frac{-b}{a}$ is solution of (E).
2. Let f a function defined and derivable on IR. Prove that if f is a solution of (E) then (f–u) is a solution of the differential equation $y' = ay$.
3. Deduce all the solutions of (E).

A cyclist runs on a straight down road. Let $v(t)$ its speed at time t, where t is expressed in seconds and $v(t)$ in meter per seconds. We assume that the function v is a solution of the differential equation $10\,v'(t) + v(t) = 30$ on the interval $[0\,;\,+\infty[$. We consider also $v(0) = 0$.

4. Demonstrate that $v(t) = 30\left(1 - e^{\frac{-t}{10}}\right)$.
5. a) Determine the direction of variation of the function v over the interval $[0\,;\,+\infty[$.
   b) Determine the limit of the function v at $+\infty$.
6. The distance traveled by the cyclist between times $t_1$and t_2 is given by:

$$d = \int_{t_1}^{t_2} v(t)dt$$

Compute the distance traveled by the cyclist during the first 35 seconds.

Exercise4. (8 Pts)

In an orthonormal direct complex plan $(O, \vec{u}, \vec{v})$ with 2 cm graphical unit, Let us consider the points A, B, C and D having respective affix $z_A = -\sqrt{3} - i$, $z_B = 1 - i\sqrt{3}$, $z_C = \sqrt{3} + i$ et $z_D = -1 + i\sqrt{3}$

1. Give the module and an argument for each of the complex numbers z_A, z_B, z_C and z_D
2. Draw the points A, B, C and D.
3. Calculate the midpoint of [AC], and the midpoint of [BD].
4. Calculate $\frac{z_B}{z_A}$. Deduce the nature of the quad ABCD

Exercise 5. (10 Pts)

In the space represent in an orthonormal $(O; \vec{i}, \vec{j}, \vec{k})$ Consider the points A(1,2,3), B(0,1,4), C(-1,-3,2), D(4,-2,5) and the vector $\vec{n}$ (2,-1,1).

1. Prove that the points A, B, C are not aligned.
2. Prove that $\vec{n}$ is a normal vector to the plane (ABC).
3. Give an equation of the plane (ABC)
4. Let (Δ) the line whose parametric representation is: $\begin{cases} x = 2 - 2t \\ y = -1 + t \\ z = 4 - t \end{cases}$ with $t \in IR$.

   Prove that the point D belongs to the line (Δ) and this line is perpendicular to the plane (ABC).
5. Let E be the orthogonal projection of point D on the plane (ABC). Show that E is the center of gravity of the triangle ABC.

Exercise 6. (4 Pts)

The distance between Sidon and Beirut is 45 km. The distance between Beirut and Tripoli is 90 km. Two cars (A and B) part simultaneously from Sidon and Beirut and reach Tripoli at the same time.

1. Give the relation between the speeds of A and B.
2. Give the speed of each car if the duration is 2 hours.

Exercise 7. (4 Pts)

The sum of two numbers is 12, their product is 96. Compute these two numbers. Explain.